Accurate Prediction of Worst Case Eye Diagrams for Non-Linear Signaling Systems

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Overview of this talk

• The Worst Case (WC) eye diagram problem
  – Starting from the basics, i.e., what is an eye diagram?
• Existing algorithms for WC eye estimation
  – PDA, illustrated with an example
• Where PDA fails
  – Cannot handle general formulations of problem
• A new algorithm for WC eye computation
  – Illustrated with an example
• Results
  – 8b/10b encoder (PCI Express, USB, etc.)
  – Our technique is much less pessimistic than PDA
What is an Eye Diagram (1/2)?

Bits → Analog Channel → Analogish “Bits”
(delay, ISI, crosstalk, etc.)

SPICE Simulation of Channel

Time

Voltage

Voltage
What is an Eye Diagram (2/2)?

Eye Diagram via SPICE Simulation of Channel

Output Voltage (V)

Time (ps)

SPICE Simulation of Channel

Eye

WC Eye
The Worst Case Eye Problem

Digital System

Analog (LTI) Channel

• Pure analog → PDA
• Analog + Digital
  – Non-Linear System
  – Correlated bits
  – PDA too pessimistic
  – Our new algorithm!

Problem: Compute worst-case eye
Peak Distortion Analysis (PDA)

- Assume channel is LTI
- Key idea: WC Eye = 2 Optimization Problems

\[
\sum_{k=-\infty}^{+\infty} b_k u(t - kT)
\]

\[
\begin{align*}
\text{WC1} & : \text{minimize } \sum_{k=-\infty}^{[\Delta/T]} b_k h(\Delta - kT) \\
\text{(subj. to } b_0 = 1) & \quad \sum_{k=-M}^{b_k} h(\Delta - kT)
\end{align*}
\]

Correlated bits: PDA fails!

\[
\begin{align*}
b_0 &= 1 \\
b_{-3} &= b_{-1} = 0 \\
b_{-2} &= b_1 = 1
\end{align*}
\]

Need mutually independent bits \([0, 1, 0, 1, 1]\)

\[
\text{WC1}(\Delta) = 0.5
\]
FSMs for Modeling Correlated Bits

• Finite number of states
• Arcs denoting state transitions
  – Each arc has an output bit

For example, this FSM can never produce the sequence [0, 1, 1]
Algorithm for Correlated WC Eye

Example. Minimize $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1$

subj. to $b_0 = 1$, $\{b_k\}$ comes from FSM

Key idea: Best partial sum ending in state $S_i$
Algorithm for Correlated WC Eye

Example. Minimize \( \frac{1}{5} b_{-3} - \frac{1}{4} b_{-2} + \frac{1}{2} b_{-1} + b_0 - \frac{1}{4} b_1 \)

subj. to \( b_0 = 1 \), \( \{b_k\} \) comes from FSM

Key idea: Best partial sum ending in state \( S_i \)

\[
\begin{align*}
S1 & \\
& 1 \rightarrow 0 \rightarrow 1
\end{align*}
\]

\[
\begin{align*}
S2 & \\
& 0 \rightarrow 1 \rightarrow 0
\end{align*}
\]

\[
\begin{align*}
S3 & \\
& 1 \rightarrow 0 \rightarrow 1
\end{align*}
\]

Best \( \frac{1}{5} b_{-3} - \frac{1}{4} b_{-2} \)
Algorithm for Correlated WC Eye

Example. Minimize \( \frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1 \)

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Key idea: Best partial sum ending in state Si

Best \( \frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 \)
Algorithm for Correlated WC Eye

Example. Minimize $\frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1$

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subj. to \( b_0 = 1, \{b_k\} \) comes from FSM

Key idea: Best partial sum ending in state \( S_i \)

Dynamic programming

Best \( \frac{1}{5}b_{-3} - \frac{1}{4}b_{-2} + \frac{1}{2}b_{-1} + b_0 - \frac{1}{4}b_1 \)

\( = \text{WC1}(\Delta) = \frac{3}{4} = 0.75 \)

Compare to PDA, which pessimistically predicts 0.5
Results: 8b/10b Encoder (1/2)

- 8b/10b Encoder + LTI Channel

8b/10b Encoder

8b parallel

10b serial

LTI Channel

8 paths (each with 3 middle states)

32 paths (each with 5 middle states)

2x, 4x, 320x, 96x

Total: 422 states
Results: 8b/10b Encoder (2/2)

- 8b/10b Encoder + LTI Channel
Summary

• WC eye computation is important
• Traditional PDA cannot handle bit correlations
• Our new technique can
• Key ideas behind our technique
  – Model bit correlations as FSMs
  – Reduce WC eye computation to an optimization problem
  – Use dynamic programming to solve the above efficiently
• Results
  – (7, 4) Hamming code
  – 8b/10b Encoder
• Future work
  – Deterministic worst case → Probabilistic distributions
Questions